Summation Notation

Tamara Kucherenko
A standard way of writing sums in compact form uses the Greek letter \( \Sigma \) (sigma).

**Definition**

If \( a_m, a_{m+1}, \ldots, a_n \) are real numbers and \( m \) and \( n \) are integers such that \( m \leq n \), then

\[
\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \ldots + a_n
\]

The letter \( \Sigma \) stands for the sum and the notation \( \sum_{i=m}^{n} \) tells us to start the summation at \( i = m \) and end it at \( i = n \).

The letter \( i \) is called the index of summation. Any other letter can be used as the index of the summation.

\( a_i \) is an expression depending on \( i \) which is called the general term of summation.
Summation Notation

Examples

1. \( \sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5 = 15 \)

2. \( \sum_{j=4}^{6} j^2 = 4^2 + 5^2 + 6^2 = 77 \)

3. \( \sum_{k=2}^{8} \frac{1}{k} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = \frac{1443}{840} \)

4. \( \sum_{i=1}^{7} 2 = 2 + 2 + 2 + 2 + 2 + 2 + 2 = 14 \)

5. Write in \( \Sigma \)-notation \( \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} = \sum_{i=3}^{9} \sqrt{i} = \sum_{j=1}^{7} \sqrt{j + 2} \)
Examples

\[ \sum_{i=1}^{n} 1 = 1 + 1 + 1 + \ldots + 1 = n \]

\[ \sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n \]

Gauss summation story:

\[ 1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100 = 101 \cdot 50 = 5050 \]
Examples

\[ \sum_{i=1}^{n} 1 = 1 + 1 + 1 + \ldots + 1 = n \]

\[ \sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \]

\[ 1 + 2 + 3 + 4 + \ldots + (n-2) + (n-1) + n = (n + 1) \cdot \frac{n}{2} \]
Summation Notation

Summation Rules

**Summation Formulas**

1. \[ \sum_{i=1}^{n} 1 = n \]
2. \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]
3. \[ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]
4. \[ \sum_{i=1}^{n} i^3 = \left( \frac{n(n+1)}{2} \right)^2 \]

**Summation Rules**

1. \[ \sum_{i=m}^{n} ca_i = c \sum_{i=m}^{n} a_i \quad (c \text{ is a constant, does not depend on } i). \]
2. \[ \sum_{i=m}^{n} (a_i \pm b_i) = \sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i \]
Evaluating Summations

**Summation Formulas**

1. \[ \sum_{i=1}^{n} 1 = n \]
2. \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]
3. \[ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]
4. \[ \sum_{i=1}^{n} i^3 = \left[ \frac{n(n+1)}{2} \right]^2 \]

Find the value of the sum

1. \[ \sum_{i=1}^{n} (7 - 5i) = \sum_{i=1}^{n} 7 - \sum_{i=1}^{n} 5i = 7 \sum_{i=1}^{n} 1 - 5 \sum_{i=1}^{n} i = 7n - 5 \frac{n(n+1)}{2} = \frac{9n-5n^2}{2} \]

2. \[ \sum_{i=1}^{n} (3+i)^2 = \sum_{i=1}^{n} (9+6i+i^2) = 9 \sum_{i=1}^{n} 1 + 6 \sum_{i=1}^{n} i + \sum_{i=1}^{n} i^2 = 9n + 6 \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6} \]

3. \[ \sum_{i=1}^{n} (i^3 - i) = \sum_{i=1}^{n} i^3 - \sum_{i=1}^{n} i = \left[ \frac{n(n+1)}{2} \right]^2 - \frac{n(n+1)}{2} \]
THE END