Functions and Their Representations

Tamara Kucherenko

Lesson Plan

- What is a function? GO
- Different ways to represent a function GO
- Even and odd functions GO
- Piecewise defined functions GO

Definition of a function

Definition

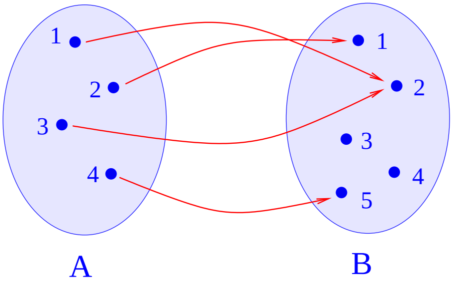
A function f is a rule that assigns to each element x in a set A exactly one element $y = f(x)$ in a set B.

Then

- A is the *domain* of f
- x is an *independent variable*
- y is a *dependent variable*
- The *range* of f is the set of all values $f(x)$ when x is in A

Typically, A is a subset of real numbers and B is all real numbers.

Arrow diagram for f



$f : A \to B$

$A = \{1, 2, 3, 4\}, \;\; B = \{1, 2, 3, 4, 5\}$
$f(1) = 2, \; f(2) = 1, \; f(3) = 2, \; f(4) = 5$

Domain of f is $\{1, 2, 3, 4\}$;
Range of f is $\{1, 2, 5\}$.

Table of values:

x	$f(x)$
1	2
2	1
3	2
4	5

Formula:

$f(x) = x^2 - 4x + 5$

Representations of a Function

We can represent a function in several different ways:

- Verbally (by a description in words)
- Numerically (by a table of values)
- Visually (by a graph)
- Algebraically (by an explicit formula)

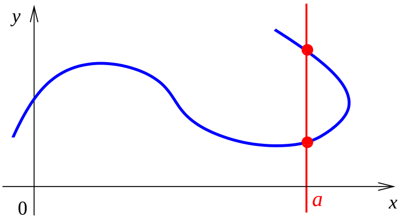
It is useful to have different representations of the same function.

Graphical Representation

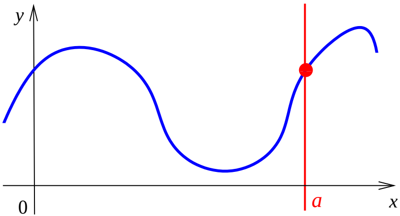
Not every curve in the *xy*-plane is a graphical representation of a function.

The Vertical Line Test

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.



Curve does not represent a function.



Curve represents a function.

Representation by a Formula

The same function could be represented by different formulas. For example, $f(x) = x^2 - 1$ and $g(x) = (x-1)(x+1)$ define the same function.

Suppose a function is defined algebraically and its domain is not specified. Then we assume that the domain is the set of all real numbers for which the formula makes sense. This set is referred to as the natural domain of a function.

$f(x) = 2x + 1$	Domain is all real numbers
$f(x) = \frac{1}{x-2}$	Domain is all real numbers except 2
$f(x) = \sqrt{x}$	Domain is all non-negative real numbers

The most common reasons for limited domains are

- the division by zero issue
- negative numbers under even radicals

Example 1

Find the domain of $f(x) = \frac{5x+3}{2x+1}$

The denominator cannot be zero, so we set

$$\begin{aligned} 2x + 1 &\neq 0 \\ 2x &\neq -1 \\ x &\neq -\frac{1}{2} \end{aligned}$$

Domain is $\boxed{(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)}$

Example 2

Find the domain of $f(x) = \sqrt{x-2}$

The expression under the radical has to be nonnegative, so we set

$$\begin{aligned} x - 2 &\geq 0 \\ x &\geq 2 \end{aligned}$$

Domain is $\boxed{[2, \infty)}$

Example 3

Find the domain of $f(x) = \frac{7x+11}{x^2-x-6}$

The denominator cannot be zero, so we set

$$x^2 - x - 6 \neq 0$$
$$(x+2)(x-3) \neq 0$$
$$x \neq -2 \; and \; x \neq 3$$

Domain is $\boxed{(-\infty, -2) \cup (-2, 3) \cup (3, \infty)}$

Example 4

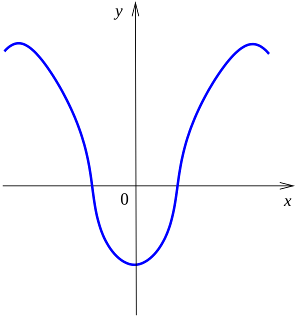
Find the domain of $f(x) = \sqrt[3]{x-5} + 3x - 1$

Since we can take a cubic root of any number, there are no restrictions on x.

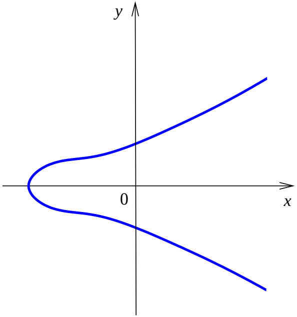
Domain is $\boxed{(-\infty, \infty)}$

Symmetry

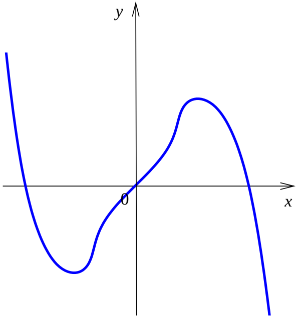
Symmetry about the y-axis



Symmetry about the x-axis

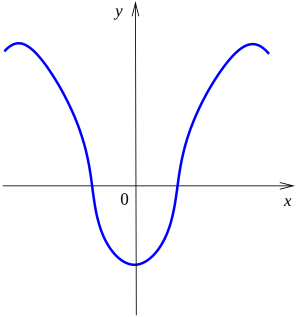


Symmetry about the origin

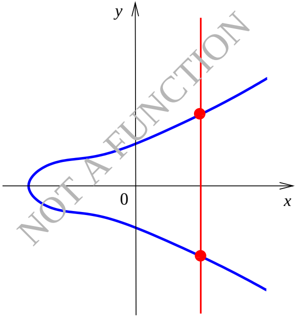


Symmetry

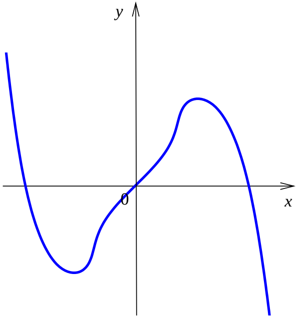
Symmetry about the y-axis



Symmetry about the x-axis



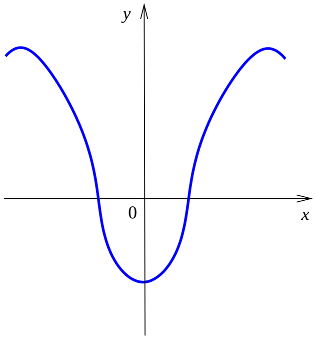
Symmetry about the origin



Apply the vertical line test

Symmetry

Symmetry about the y-axis



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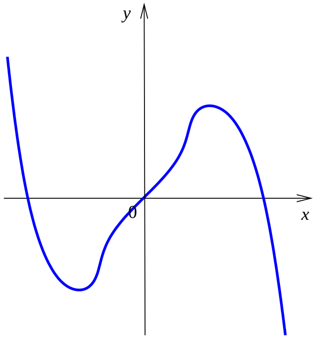
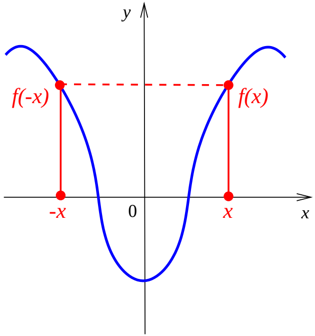


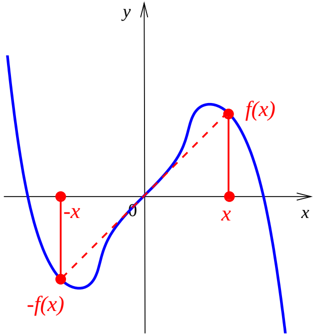
Symmetry

Symmetry about the y-axis



f is an even function if $f(-x) = f(x)$ for every x in its domain

Symmetry about the origin



f is an odd function if $f(-x) = -f(x)$ for every x in its domain

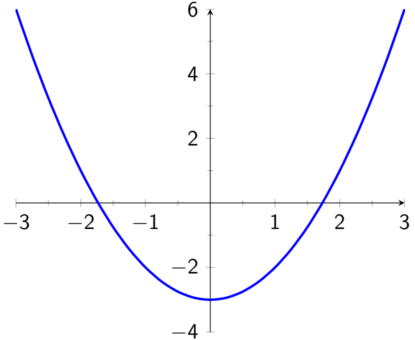
Example 1

Determine whether $f(x) = x^2 - 3$ is even, odd, or neither even or odd

We plug in $-x$ instead of x:

$$f(-x) = (-x)^2 - 3 = x^2 - 3 = f(x)$$

Therefore, f is $\boxed{even}$.



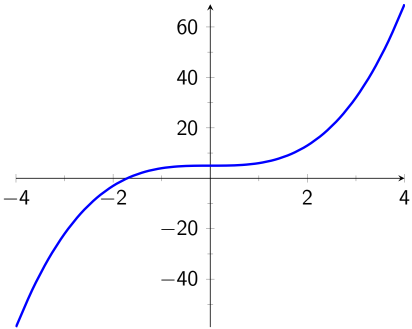
Example 2

Determine whether $f(x) = x^3 + 5$ is even, odd, or neither even or odd

We plug in $-x$ instead of x:

$$\begin{aligned} f(-x) &= (-x)^3 + 5 \\ &= -x^3 + 5 \neq f(x) \\ &= -(x^3 - 5) \neq -f(x) \end{aligned}$$

Therefore, f is $\boxed{\textit{neither}}$.



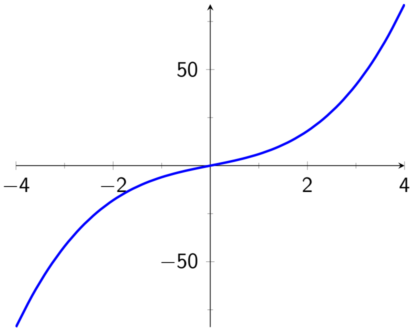
Example 3

Determine whether $f(x) = x^3 + 5x$ is even, odd, or neither even or odd

We plug in $-x$ instead of x:

$$\begin{aligned} f(-x) &= (-x)^3 + 5(-x) \\ &= -x^3 - 5x \\ &= -(x^3 + 5x) = -f(x) \end{aligned}$$

Therefore, f is $\boxed{odd}$.



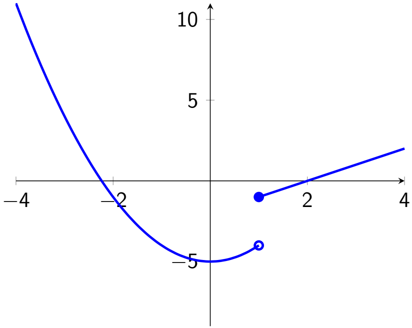
Piecewise Defined Functions

If a function f is defined by different formulas in different parts of its domain, we say that f is piecewise defined.

For example,

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x < 1 \\ x - 2 & \text{if } x \geq 1 \end{cases}$$

$f(-1) = (-1)^2 - 5 = -4$
$f(2) = 2 - 2 = 0$
$f(1) = 1 - 2 = -1$



The Absolute Value Function

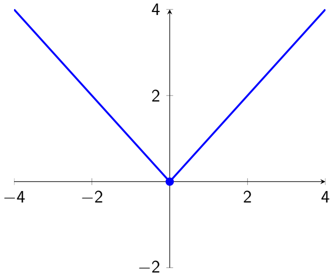
The absolute value of a number x, denoted by $|x|$, is the distance from 0 to x on the real number line. Distances are always positive or 0, so we have $|x| \geq 0$.

For example,

$|2| = 2, \quad |-5| = 5, \quad |0| = 0$

The Absolute Value

$$|x| = \begin{cases} -x & \textit{if } x < 0 \\ x & \textit{if } x \geq 0 \end{cases}$$



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