

# Limit Laws

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Assume that  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  exist. Then:

- **Sum/Difference Law:**  $\lim_{x \rightarrow a} (f(x) \pm g(x)) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
- **Constant Multiple Law:** For any number  $k$ ,  $\lim_{x \rightarrow a} kf(x) = k \lim_{x \rightarrow a} f(x)$
- **Product Law:**  $\lim_{x \rightarrow a} f(x)g(x) = \left( \lim_{x \rightarrow a} f(x) \right) \left( \lim_{x \rightarrow a} g(x) \right)$
- **Quotient Law:** If  $\lim_{x \rightarrow a} g(x) \neq 0$ , then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

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For any numbers  $a$  and  $c$ ,  $\lim_{x \rightarrow a} c = c$  and  $\lim_{x \rightarrow a} x = a$

**Example 1.** Use the Limit Laws to evaluate  $\lim_{x \rightarrow 4} x^2$ .

Product Law:

$$\lim_{x \rightarrow 4} x^2 = \left( \lim_{x \rightarrow 4} x \right) \cdot \left( \lim_{x \rightarrow 4} x \right) = 4 \cdot 4 = 16$$

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**Example 2.** Use the Limit Laws to evaluate  $\lim_{x \rightarrow 2} (2x^3 - 6x + 4)$ .

$$\begin{aligned} \lim_{x \rightarrow 2} (2x^3 - 6x + 4) &= \lim_{x \rightarrow 2} (2x^3) - \lim_{x \rightarrow 2} (6x) + \lim_{x \rightarrow 2} 4 \\ &= 2(\lim_{x \rightarrow 2} x)(\lim_{x \rightarrow 2} x)(\lim_{x \rightarrow 2} x) - 6 \lim_{x \rightarrow 2} x + \lim_{x \rightarrow 2} 4 \\ &= 2(2)^3 - 6(2) + 4 = \boxed{8} \end{aligned}$$

**Example 3.** Use the Limit Laws to evaluate  $\lim_{x \rightarrow 1} \frac{x-5}{2x}$ .

$$\lim_{x \rightarrow 1} \frac{x-5}{2x} = \frac{\lim_{x \rightarrow 1} (x-5)}{\lim_{x \rightarrow 1} 2x} = \frac{\lim_{x \rightarrow 1} x - \lim_{x \rightarrow 1} 5}{2 \lim_{x \rightarrow 1} x} = \frac{1-5}{2 \cdot 1} = \boxed{-2}$$