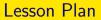
The Limit of a Function

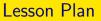
Tamara Kucherenko

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• Numerical and Graphical Investigation

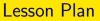




- Numerical and Graphical Investigation
- Informal Definition of a Limit



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- Numerical and Graphical Investigation
- Informal Definition of a Limit
- One-Sided Limits



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Lesson Plan

- Numerical and Graphical Investigation
- Informal Definition of a Limit
- One-Sided Limits
- Infinite limits

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Lesson Plan

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The limit of a function is a fundamental concept concerning the behavior of that function near a particular input.



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Investigate the behavior of the function $f(x) = \frac{\sin x}{x}$ for values of x near 0.



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The domain of f is $(-\infty, 0) \cup (0, \infty)$, f(0) is undefined.

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A Numerical and Graphical approach

The limit of a function is a fundamental concept concerning the behavior of that function near a particular input.

Investigate the behavior of the function $f(x) = \frac{\sin x}{x}$ for values of x near 0.

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A Numerical and Graphical approach

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Investigate the behavior of the function $f(x) = \frac{\sin x}{x}$ for values of x near 0.

The domain of f is $(-\infty, 0) \cup (0, \infty)$, f(0) is undefined.

We can compute f(x) for values of x as close to 0 as we like.

To indicate that x takes on values (both positive and negative) that get closer and closer to 0 we write $x \to 0$.

 $f(x) = \frac{\sin x}{x}$ for x near 0

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
-1.000	0.8414709	1.000	0.8414709
-0.500	0. 9 588510	0.500	0. 9 588510
-0.100	0. 99 83341	0.100	0. 99 83341
-0.050	0. 999 5833	0.050	0. 999 5833
-0.010	0. 9999 833	0.010	0. 9999 833
-0.005	0. 99999 58	0.005	0. 99999 58
-0.001	0. 999999 8	0.001	0. 999999 8

Tamara Kucherenko

One-Sided Limits

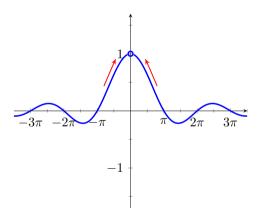
The Limit of a Function

Infinite Limits

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 $f(x) = \frac{\sin x}{x}$ for x near 0

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
-1.000	0.8414709	1.000	0.8414709
-0.500	0. 9 588510	0.500	0. 9 588510
-0.100	0. 99 83341	0.100	0. 99 83341
-0.050	0. 999 5833	0.050	0. 999 5833
-0.010	0. 9999 833	0.010	0. 9999 833
-0.005	0. 99999 58	0.005	0. 99999 58
-0.001	0. 999999 8	0.001	0. 999999 8



Infinite Limits

x

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
-1.000	0.8414709	1.000	0.8414709
-0.500	0. 9 588510	0.500	0. 9 588510
-0.100	0. 99 83341	0.100	0. 99 83341
-0.050	0. 999 5833	0.050	0. 999 5833
-0.010	0. 9999 833	0.010	0. 9999 833
-0.005	0. 99999 58	0.005	0. 99999 58
-0.001	0. 999999 8	0.001	0. 999999 8

f(x) converges to 1 as x approaches zero or $\lim_{x \to 0} f(x) = 1$.

$$-3\pi - 2\pi - \pi - 1$$

$$f(x) = \frac{\sin x}{x}$$
 for x near

One-Sided Limits

Infinite Limits

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Informal Definition of a Limit

Tamara Kucherenko The Limit of a Function

One-Sided Limits

Infinite Limits

Informal Definition of a Limit

Definition

Suppose f(x) is defined for all x near the number a.

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Informal Definition of a Limit

Definition

Suppose f(x) is defined for all x near the number a. We say that

$$\lim_{x \to a} f(x) = L$$

if we can make the values of f(x) as close to L as we like by taking x to be any number sufficiently close (but not equal) to a.

Informal Definition of a Limit

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An alternative notation is $f(x) \rightarrow L$ as $x \rightarrow a$.

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If the values of f(x) do not approach any number as $x \to a$, we say that $\lim_{x\to a} f(x)$ does not exist.

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If the values of f(x) do not approach any number as $x \to a$, we say that $\lim_{x\to a} f(x)$ does not exist.

If f(x) approaches a limit as $x \to a$, then the limiting value L is unique.

One-Sided Limits

Infinite Limits

Example 1

Find $\lim_{x \to 4} \frac{x-4}{\sqrt{x-2}}$.



One-Sided Limits

Infinite Limits

Example 1

Find
$$\lim_{x \to 4} \frac{x-4}{\sqrt{x-2}}$$
. Notice that $\frac{4-4}{\sqrt{4-2}} = \frac{0}{0}$ (undefined)



One-Sided Limits

Infinite Limits

Example 1

Find
$$\lim_{x \to 4} \frac{x-4}{\sqrt{x-2}}$$
. Notice that $\frac{4-4}{\sqrt{4-2}} = \frac{0}{0}$ (undefined)

x	$\frac{x-4}{\sqrt{x-2}}$	x	$\frac{x-4}{\sqrt{x-2}}$
3.90000	3.9 74841	4.10000	4.0 24845
3.99000	3.99 7498	4.01000	4.00 2498
3.99900	3.999 750	4.00100	4.000 250
3.99990	3.9999 75	4.00010	4.0000 25
3.99999	3.99999 7	4.00001	4.00000 2

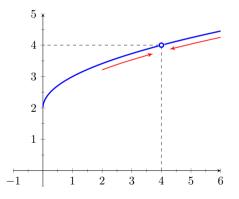
Infinite Limits

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Example 1

Find
$$\lim_{x
ightarrow 4}rac{x-4}{\sqrt{x-2}}$$
. Notice that $rac{4-4}{\sqrt{4-2}}=rac{0}{0}$ (undefined)

x	$\frac{x-4}{\sqrt{x-2}}$	x	$\frac{x-4}{\sqrt{x-2}}$
3.90000	3.9 74841	4.10000	4.0 24845
3.99000	3.99 7498	4.01000	4.00 2498
3.99900	3.999 750	4.00100	4.000 250
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Infinite Limits

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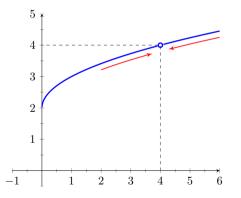
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$$\lim_{x
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x	$\frac{x-4}{\sqrt{x-2}}$	x	$\frac{x-4}{\sqrt{x-2}}$
3.90000	3.9 74841	4.10000	4.0 24845
3.99000	3.99 7498	4.01000	4.00 2498
3.99900	3.999 750	4.00100	4.000 250
3.99990	3.9999 75	4.00010	4.0000 25
3.99999	3.99999 7	4.00001	4.00000 2

Therefore,

$$\lim_{x \to 4} \frac{x-4}{\sqrt{x-2}} = 4.$$



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Example 2

One-Sided Limits

Infinite Limits

Find $\lim_{x \to 3} x^2$.



Example 2

Infinite Limits

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Find $\lim_{x \to 3} x^2$.

x	x^2	x	x^2
2.90000	8 .410000	3.10000	9 .610000
2.99000	8.9 40100	3.01000	9.0 60100
2.99900	8.99 4001	3.00100	9.00 6001
2.99990	8.999 400	3.00010	9.000 600
2.99999	8.9999 40	3.00001	9.0000 60

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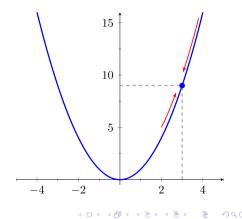
Example 2

One-Sided Limits

Infinite Limits

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2.99990	8.999 400	3.00010	9.000 600
2.99999	8.9999 40	3.00001	9.0000 60



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Example 2

One-Sided Limits

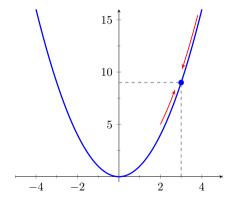
Infinite Limits

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2.99900	8.99 4001	3.00100	9.00 6001
2.99990	8.999 400	3.00010	9.000 600
2.99999	8.9999 40	3.00001	9.0000 60

Therefore,
$$\lim_{x \to 3} x^2 = 9.$$



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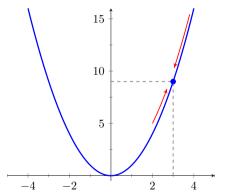
Example 2

Find $\lim_{x \to 3} x^2$.

x	x^2	x	x^2
2.90000	8 .410000	3.10000	9 .610000
2.99000	8.9 40100	3.01000	9.0 60100
2.99900	8.99 4001	3.00100	9.00 6001
2.99990	8.999 400	3.00010	9.000 600
2.99999	8.9999 40	3.00001	9.0000 60

Therefore,
$$\lim_{x \to 3} x^2 = 9.$$

Here $f(x) = x^2$ is defined at x = 3 and f(3) = 9,



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Infinite Limits

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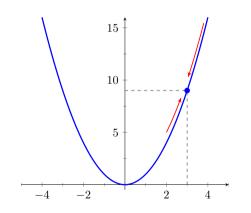
Example 2

Find $\lim_{x \to 3} x^2$.

x	x^2	x	x^2
2.90000	8 .410000	3.10000	9 .610000
2.99000	8.9 40100	3.01000	9.0 60100
2.99900	8.99 4001	3.00100	9.00 6001
2.99990	8.999 400	3.00010	9.000 600
2.99999	8.9999 40	3.00001	9.0000 60

Therefore,
$$\lim_{x \to 3} x^2 = 9.$$

Here $f(x) = x^2$ is defined at x = 3 and f(3) = 9, so the limit is equal to the function value.



Infinite Limits

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Example 3

One-Sided Limits

Infinite Limits

Find $\lim_{x\to 0} \sin \frac{\pi}{x}$.

Example 3

One-Sided Limits

Infinite Limits

Find $\lim_{x\to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at x = 0.



Example 3

Find $\lim_{x\to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at x = 0.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

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Example 3

Find $\lim_{x\to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at x = 0.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

We guess that $\lim_{x \to 0} \sin \frac{\pi}{x} = 0.$

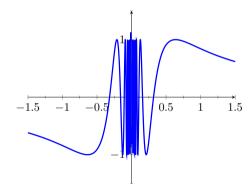
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Example 3

Find
$$\lim_{x\to 0} \sin \frac{\pi}{x}$$
. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

We guess that
$$\lim_{x \to 0} \sin \frac{\pi}{x} = 0.$$



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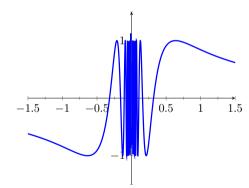
Infinite Limits

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x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

We guess that $\lim_{x\to 0} \sin \frac{\pi}{x} \neq 0.$



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Infinite Limits

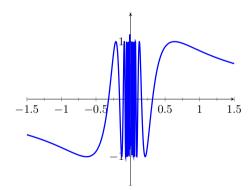
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Find
$$\lim_{x\to 0} \sin \frac{\pi}{x}$$
. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

We guess that $\lim_{x\to 0} \frac{\pi}{x} \neq 0.$ <u>Reason:</u> $\sin \frac{\pi}{0.01} = \sin(100\pi) = 0$, since $\sin \pi n = 0.$



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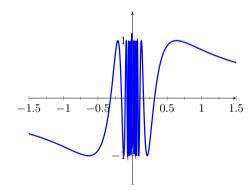
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Find $\lim_{x\to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at x = 0.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
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-0.00001	0.000000	0.00001	0.000000

We guess that
$$\lim_{x\to 0} \sin \frac{\pi}{x} \neq 0$$
.
Reason: $\sin \frac{\pi}{0.01} = \sin(100\pi) = 0$, since $\sin \pi n = 0$.
However, $\sin(\pi n + \frac{\pi}{2}) = \pm 1$.



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Infinite Limits

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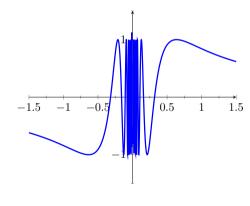
Infinite Limits

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x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
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-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

We guess that
$$\lim_{x\to 0} \sin \frac{\pi}{x} \neq 0$$
.
Reason: $\sin \frac{\pi}{0.01} = \sin(100\pi) = 0$, since $\sin \pi n = 0$.
However, $\sin(\pi n + \frac{\pi}{2}) = \pm 1$. Solving $\pi n + \frac{\pi}{2} = \frac{\pi}{x}$ for x
with $n = 100$ gives $x = 0.00995$ and $\sin \frac{\pi}{0.00995} = 0.997$



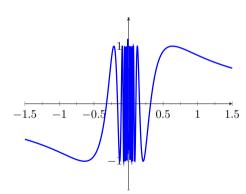
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Example 3

Find $\lim_{x\to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at x = 0.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.49990	-0.0012569	0.50000	0.0000000
-0.02090	0.4626743	0.02100	-0.9308737
-0.00985	0.9974262	0.00995	0.9999688
-0.00063	0.8119380	0.00073	-0.4171936
-0.00011	-0.2817326	0.00011	0.2817326



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Infinite Limits

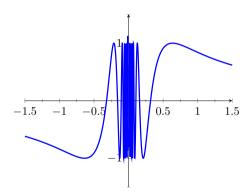
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Example 3

Find
$$\lim_{x\to 0} \sin \frac{\pi}{x}$$
. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.49990	-0.0012569	0.50000	0.0000000
-0.02090	0.4626743	0.02100	-0.9308737
-0.00985	0.9974262	0.00995	0.9999688
-0.00063	0.8119380	0.00073	-0.4171936
-0.00011	-0.2817326	0.00011	0.2817326

Therefore,
$$\lim_{x \to 0} \sin \frac{\pi}{x}$$
 does not exist.



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Infinite Limits

One-Sided Limits

One-Sided Limits

Infinite Limits

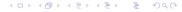
For a number a we write

• $x \to a^-$ if x approaches a from the left (through values less than a)

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For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)



One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)



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One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)
- We say that $\lim_{x\to a^-} f(x) = L$ (left-hand limit) if we can make the values of f(x) as close to L as we like by taking x to be any number smaller than a and sufficiently close to a.

One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)
- We say that $\lim_{x\to a^-} f(x) = L$ (left-hand limit) if we can make the values of f(x) as close to L as we like by taking x to be any number smaller than a and sufficiently close to a.

Similarly, $\lim_{x \to a^+} f(x) = L$ (right-hand limit) if we can make the values of f(x) as close to L as we like by taking x to be any number greater than a and sufficiently close to a.

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One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)

We say that $\lim_{x\to a^-} f(x) = L$ (left-hand limit) if we can make the values of f(x) as close to L as we like by taking x to be any number smaller than a and sufficiently close to a.

Similarly, $\lim_{x \to a^+} f(x) = L$ (right-hand limit) if we can make the values of f(x) as close to L as we like by taking x to be any number greater than a and sufficiently close to a.

$$\lim_{x \to a} f(x) = L \text{ if and only if } \lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L$$

Example 1

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For the function

Example 1

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2\\ 9 - 2x & \text{if } x \ge 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1
$$\lim_{x \to 2^{-}} f(x)$$

2 $\lim_{x \to 2^{+}} f(x)$
3 $\lim_{x \to 1} f(x)$

$$\lim_{x \to 2} f(x)$$

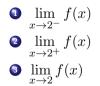
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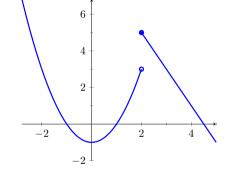
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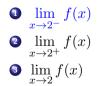
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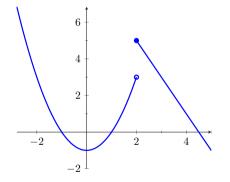
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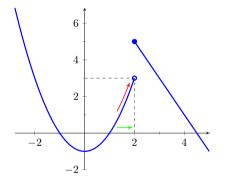
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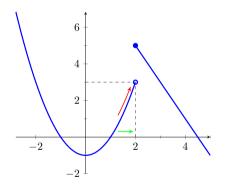
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$$\lim_{x \to 2^{-}} f(x) = 3$$

$$\lim_{x \to 2^{+}} f(x)$$

$$\lim_{x \to 2} f(x)$$



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Infinite Limits

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For the function

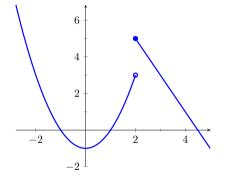
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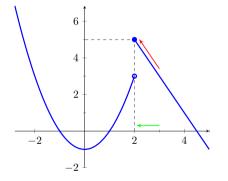
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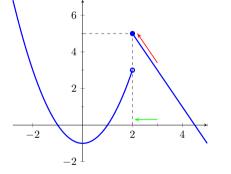
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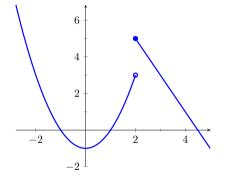
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$$\lim_{x \to 2^{-}} f(x) = 3$$

2
$$\lim_{x \to 2^+} f(x) = 5$$

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For the function

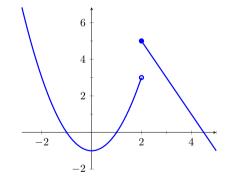
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$$\lim_{x \to 2^-} f(x) = 3$$

2
$$\lim_{x \to 2^+} f(x) = 5$$

 $\textcircled{O} \ \lim_{x \to 2^+} f(x) \ \mathsf{DNE} \text{, since } \lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$



Example 2

One-Sided Limits

Infinite Limits





Example 2

Investigate
$$\lim_{x \to 0} \frac{|x|}{x}$$
.
Recall that $|x| = \begin{cases} -x & \text{if } x < 0\\ x & \text{if } x \ge 0 \end{cases}$

Tamara Kucherenko The Limit of a Function

One-Sided Limits

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Infinite Limits

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Example 2

Investigate
$$\lim_{x\to 0} \frac{|x|}{x}$$
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Recall that $|x| = \begin{cases} -x & \text{if } x < 0\\ x & \text{if } x \ge 0 \end{cases}$
When $x < 0$, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$.

Tamara Kucherenko The Limit of a Function

One-Sided Limits

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Infinite Limits

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Example 2

Investigate $\lim_{x\to 0} \frac{|x|}{x}$. Recall that $|x| = \begin{cases} -x & \text{if } x < 0\\ x & \text{if } x \ge 0 \end{cases}$ When x < 0, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$. When x > 0, we have $\frac{|x|}{x} = \frac{x}{x} = 1$.

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One-Sided Limits

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Example 2

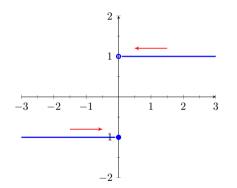
Investigate $\lim_{x \to 0} \frac{|x|}{x}$. Recall that $|x| = \begin{cases} -x & \text{if } x < 0\\ x & \text{if } x \ge 0 \end{cases}$ When x < 0, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$. When x > 0, we have $\frac{|x|}{x} = \frac{x}{x} = 1$. Therefore, $\frac{|x|}{x} = \begin{cases} -1 & \text{if } x < 0\\ 1 & \text{if } x > 0 \end{cases}$ One-Sided Limits

Infinite Limits

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(a)

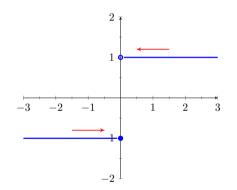
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One-Sided Limits

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Example 2

Investigate $\lim_{x \to 0} \frac{|x|}{x}$. Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$ When x < 0, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$. When x > 0, we have $\frac{|x|}{x} = \frac{x}{x} = 1$. Therefore, $\frac{|x|}{x} = \begin{cases} -1 & \text{if } x < 0\\ 1 & \text{if } x > 0 \end{cases}$ $\lim_{x \to 0^{-}} \frac{|x|}{x} = -1 & \lim_{x \to 0^{+}} \frac{|x|}{x} = 1 \end{cases}$



(a)

One-Sided Limits

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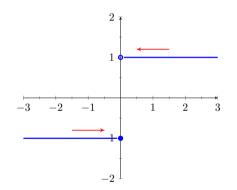
Tamara Kucherenko The Limit of a Function

Infinite Limits

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(a)

Infinite Limits

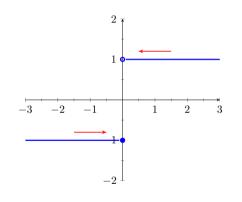
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One-Sided Limits

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One-Sided Limits

 $\lim_{x \to a} f(x) = \infty$ means that the values of f(x) can be made arbitrarily large by taking x to be any number sufficiently close (but not equal) to a.



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For example,

$$\lim_{x \to 0} \frac{1}{x^2}$$

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A Numerical and Graphical approach

Infinite Limits

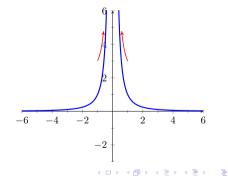
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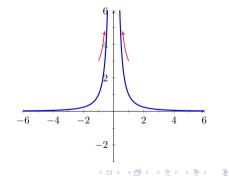
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A Numerical and Graphical approach

Infinite Limits

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One-Sided Limits

 $\lim_{x\to a} f(x) = -\infty$ means that the values of f(x) can be made arbitrarily large negative by taking x to be any number sufficiently close (but not equal) to a.



One-Sided Limits

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Example Find $\lim_{x\to 0^-} \frac{1}{x}$ and $\lim_{x\to 0^+} \frac{1}{x}$.



 $\lim_{x\to a} f(x) = -\infty$ means that the values of f(x) can be made arbitrarily large negative by taking x to be any number sufficiently close (but not equal) to a.

Example Find
$$\lim_{x\to 0^-} \frac{1}{x}$$
 and $\lim_{x\to 0^+} \frac{1}{x}$.
If x is a small negative number, $\frac{1}{x}$ is a large negative number.

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Example Find
$$\lim_{x\to 0^-} \frac{1}{x}$$
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If x is a small negative number, $\frac{1}{x}$ is a large negative number.
Therefore, $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.



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One-Sided Limits

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One-Sided Limits

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 and $\lim_{x\to 0^+} \frac{1}{x}$.
If x is a small negative number, $\frac{1}{x}$ is a large negative number.
Therefore, $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.

If x is a small positive number, $\frac{1}{x}$ is a large positive number.

Therefore,
$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$
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One-Sided Limits

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 and $\lim_{x\to 0^+} \frac{1}{x}$.
If x is a small negative number, $\frac{1}{x}$ is a large negative number.
Therefore, $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.

If x is a small positive number, $\frac{1}{x}$ is a large positive number.

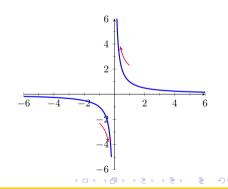
Therefore,
$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$
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One-Sided Limits

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 and $\lim_{x\to 0^{+}} \frac{1}{x}$.
If x is a small negative number, $\frac{1}{x}$ is a large negative number.
Therefore, $\lim_{x\to 0^{-}} \frac{1}{x} = -\infty$.
If x is a small positive number, $\frac{1}{x}$ is a large positive number.

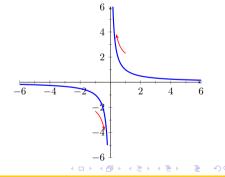
Therefore,
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One-Sided Limits

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If x is a small positive number, $\frac{1}{x}$ is a large positive number.
Therefore, $\lim_{x\to 0^{+}} \frac{1}{x} = \infty$.



Therefore,
$$\lim_{x o 0^+} rac{1}{x} = \infty.$$

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