The Limit of a Function

Tamara Kucherenko

Lesson Plan

- Numerical and Graphical Investigation

Lesson Plan

- Numerical and Graphical Investigation
- Informal Definition of a Limit

Lesson Plan

- Numerical and Graphical Investigation
- Informal Definition of a Limit
- One-Sided Limits

Lesson Plan

- Numerical and Graphical Investigation
- Informal Definition of a Limit
- One-Sided Limits
- Infinite limits

Lesson Plan

- Numerical and Graphical Investigation
- Informal Definition of a Limit
- One-Sided Limits
- Infinite limits

Lesson Plan

- Numerical and Graphical Investigation
- Informal Definition of a Limit
- One-Sided Limits
- Infinite limits

A Numerical and Graphical approach

The limit of a function is a fundamental concept concerning the behavior of that function near a particular input.

A Numerical and Graphical approach

The limit of a function is a fundamental concept concerning the behavior of that function near a particular input.

Investigate the behavior of the function $f(x) = \dfrac{\sin x}{x}$ for values of x near 0.

A Numerical and Graphical approach

The limit of a function is a fundamental concept concerning the behavior of that function near a particular input.

Investigate the behavior of the function $f(x) = \dfrac{\sin x}{x}$ for values of x near 0.

The domain of f is $(-\infty, 0) \cup (0, \infty)$, $f(0)$ is undefined.

A Numerical and Graphical approach

The limit of a function is a fundamental concept concerning the behavior of that function near a particular input.

Investigate the behavior of the function $f(x) = \frac{\sin x}{x}$ for values of x near 0.

The domain of f is $(-\infty, 0) \cup (0, \infty)$, $f(0)$ is undefined.

We can compute $f(x)$ for values of x as close to 0 as we like.

A Numerical and Graphical approach

The limit of a function is a fundamental concept concerning the behavior of that function near a particular input.

Investigate the behavior of the function $f(x) = \frac{\sin x}{x}$ for values of x near 0.

The domain of f is $(-\infty, 0) \cup (0, \infty)$, $f(0)$ is undefined.

We can compute $f(x)$ for values of x as close to 0 as we like.

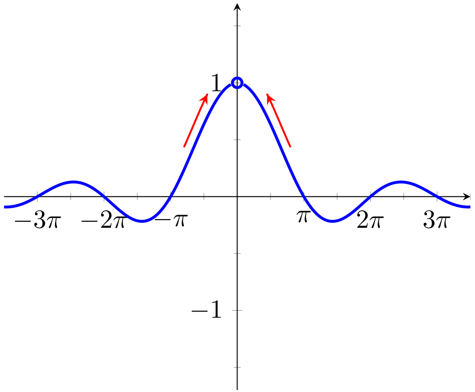
To indicate that x takes on values (both positive and negative) that get closer and closer to 0 we write $x \to 0$.

$f(x) = \frac{\sin x}{x}$ for x near 0

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
-1.000	0.8414709	1.000	0.8414709
-0.500	0.**9**588510	0.500	0.**9**588510
-0.100	0.**99**83341	0.100	0.**99**83341
-0.050	0.**999**5833	0.050	0.**999**5833
-0.010	0.**9999**833	0.010	0.**9999**833
-0.005	0.**99999**58	0.005	0.**99999**58
-0.001	0.**999999**8	0.001	0.**999999**8

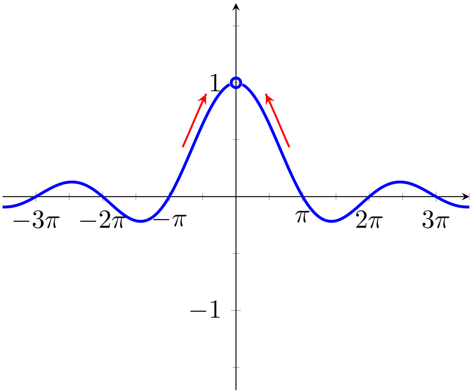
$f(x) = \frac{\sin x}{x}$ for x near 0

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
-1.000	0.8414709	1.000	0.8414709
-0.500	0.**9**588510	0.500	0.**9**588510
-0.100	0.**99**83341	0.100	0.**99**83341
-0.050	0.**999**5833	0.050	0.**999**5833
-0.010	0.**9999**833	0.010	0.**9999**833
-0.005	0.**99999**58	0.005	0.**99999**58
-0.001	0.**999999**8	0.001	0.**999999**8



$f(x) = \frac{\sin x}{x}$ for x near 0

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
-1.000	0.8414709	1.000	0.8414709
-0.500	0.**9**588510	0.500	0.**9**588510
-0.100	0.**99**83341	0.100	0.**99**83341
-0.050	0.**999**5833	0.050	0.**999**5833
-0.010	0.**9999**833	0.010	0.**9999**833
-0.005	0.**99999**58	0.005	0.**99999**58
-0.001	0.**999999**8	0.001	0.**999999**8



$f(x)$ converges to 1 as x approaches zero or $\lim_{x \to 0} f(x) = 1$.

Informal Definition of a Limit

Informal Definition of a Limit

Definition

Suppose $f(x)$ is defined for all x near the number a.

Informal Definition of a Limit

Definition

Suppose $f(x)$ is defined for all x near the number a. We say that

$$\boxed{\lim_{x \to a} f(x) = L}$$

if we can make the values of $f(x)$ as close to L as we like by taking x to be any number sufficiently close (but not equal) to a.

Informal Definition of a Limit

Definition

Suppose $f(x)$ is defined for all x near the number a.We say that

$$\boxed{\lim_{x \to a} f(x) = L}$$

if we can make the values of $f(x)$ as close to L as we like by taking x to be any number sufficiently close (but not equal) to a.

An alternative notation is $f(x) \to L$ as $x \to a$.

Informal Definition of a Limit

Definition

Suppose $f(x)$ is defined for all x near the number a. We say that

$$\boxed{\lim_{x \to a} f(x) = L}$$

if we can make the values of $f(x)$ as close to L as we like by taking x to be any number sufficiently close (but not equal) to a.

An alternative notation is $f(x) \to L$ as $x \to a$.

If the values of $f(x)$ do not approach any number as $x \to a$, we say that $\lim_{x \to a} f(x)$ does not exist.

Informal Definition of a Limit

Definition

Suppose $f(x)$ is defined for all x near the number a. We say that

$$\boxed{\lim_{x \to a} f(x) = L}$$

if we can make the values of $f(x)$ as close to L as we like by taking x to be any number sufficiently close (but not equal) to a.

An alternative notation is $f(x) \to L$ as $x \to a$.

If the values of $f(x)$ do not approach any number as $x \to a$,
we say that $\lim_{x \to a} f(x)$ does not exist.

If $f(x)$ approaches a limit as $x \to a$, then the limiting value L is unique.

Example 1

Find $\lim\limits_{x\to 4} \frac{x-4}{\sqrt{x}-2}$.

Example 1

Find $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$. Notice that $\frac{4-4}{\sqrt{4}-2} = \frac{0}{0}$ (undefined)

Example 1

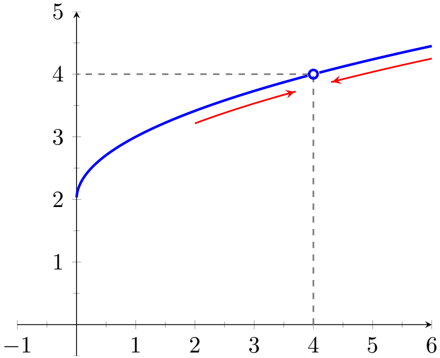
Find $\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2}$. Notice that $\frac{4-4}{\sqrt{4}-2} = \frac{0}{0}$ (undefined)

x	$\frac{x-4}{\sqrt{x}-2}$	x	$\frac{x-4}{\sqrt{x}-2}$
3.90000	**3.9**74841	4.10000	**4.0**24845
3.99000	**3.99**7498	4.01000	**4.00**2498
3.99900	**3.999**750	4.00100	**4.000**250
3.99990	**3.9999**75	4.00010	**4.0000**25
3.99999	**3.99999**7	4.00001	**4.00000**2

Example 1

Find $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$. Notice that $\frac{4-4}{\sqrt{4}-2} = \frac{0}{0}$ (undefined)

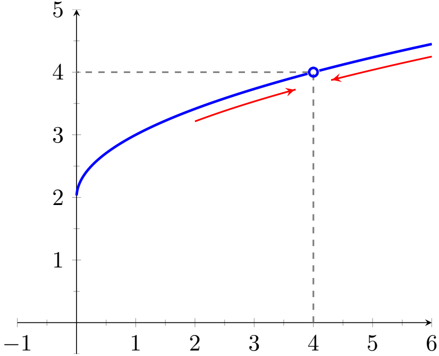
x	$\frac{x-4}{\sqrt{x}-2}$	x	$\frac{x-4}{\sqrt{x}-2}$
3.90000	**3.9**74841	4.10000	**4.0**24845
3.99000	**3.99**7498	4.01000	**4.00**2498
3.99900	**3.999**750	4.00100	**4.000**250
3.99990	**3.9999**75	4.00010	**4.0000**25
3.99999	**3.99999**7	4.00001	**4.00000**2



Example 1

Find $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$. Notice that $\frac{4-4}{\sqrt{4}-2} = \frac{0}{0}$ (undefined)

x	$\frac{x-4}{\sqrt{x}-2}$	x	$\frac{x-4}{\sqrt{x}-2}$
3.90000	**3.9**74841	4.10000	**4.0**24845
3.99000	**3.99**7498	4.01000	**4.00**2498
3.99900	**3.999**750	4.00100	**4.000**250
3.99990	**3.9999**75	4.00010	**4.0000**25
3.99999	**3.99999**7	4.00001	**4.00000**2



Therefore, $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = 4$.

Example 2

Find $\lim_{x \to 3} x^2$.

Example 2

Find $\lim_{x \to 3} x^2$.

x	x^2	x	x^2
2.90000	**8**.410000	3.10000	**9**.610000
2.99000	**8.9**40100	3.01000	**9.0**60100
2.99900	**8.99**4001	3.00100	**9.00**6001
2.99990	**8.999**400	3.00010	**9.000**600
2.99999	**8.9999**40	3.00001	**9.0000**60

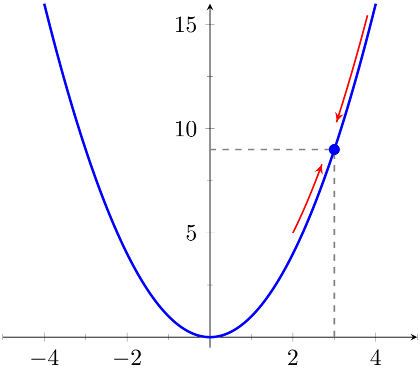
,

Example 2

Find $\lim_{x \to 3} x^2$.

x	x^2	x	x^2
2.90000	**8**.410000	3.10000	**9**.610000
2.99000	**8.9**40100	3.01000	**9.0**60100
2.99900	**8.99**4001	3.00100	**9.00**6001
2.99990	**8.999**400	3.00010	**9.000**600
2.99999	**8.9999**40	3.00001	**9.0000**60

,



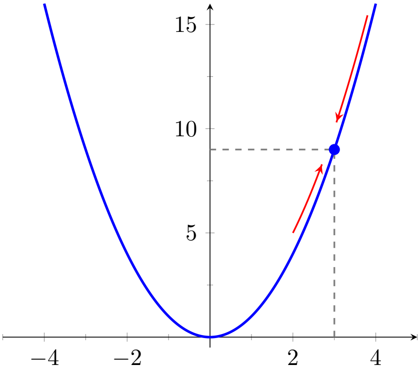
Example 2

Find $\lim_{x\to 3} x^2$.

x	x^2	x	x^2
2.90000	**8**.410000	3.10000	**9**.610000
2.99000	**8.9**40100	3.01000	**9.0**60100
2.99900	**8.99**4001	3.00100	**9.00**6001
2.99990	**8.999**400	3.00010	**9.000**600
2.99999	**8.9999**40	3.00001	**9.0000**60

Therefore, $\lim_{x\to 3} x^2 = 9$.

,



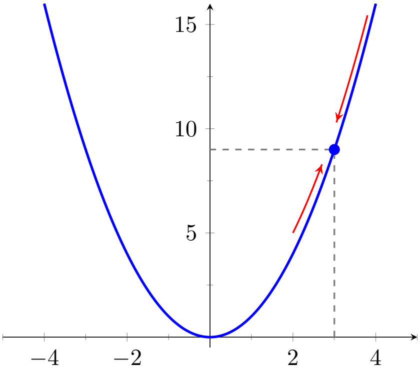
Example 2

Find $\lim_{x \to 3} x^2$.

x	x^2	x	x^2
2.90000	**8**.410000	3.10000	**9**.610000
2.99000	**8.9**40100	3.01000	**9.0**60100
2.99900	**8.99**4001	3.00100	**9.00**6001
2.99990	**8.999**400	3.00010	**9.000**600
2.99999	**8.9999**40	3.00001	**9.0000**60

Therefore, $\lim_{x \to 3} x^2 = 9$.

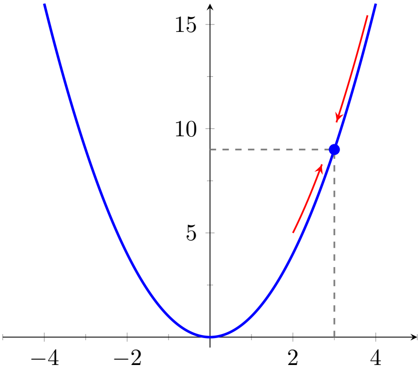
Here $f(x) = x^2$ is defined at $x = 3$ and $f(3) = 9$,



Example 2

Find $\lim_{x \to 3} x^2$.

x	x^2	x	x^2
2.90000	**8**.410000	3.10000	**9**.610000
2.99000	**8.9**40100	3.01000	**9.0**60100
2.99900	**8.99**4001	3.00100	**9.00**6001
2.99990	**8.999**400	3.00010	**9.000**600
2.99999	**8.9999**40	3.00001	**9.0000**60



Therefore, $\lim_{x \to 3} x^2 = 9$.

Here $f(x) = x^2$ is defined at $x = 3$ and $f(3) = 9$, so the limit is equal to the function value.

Example 3

Find $\lim_{x\to 0} \sin \frac{\pi}{x}$.

Example 3

Find $\lim_{x\to 0} \sin\frac{\pi}{x}$. The function $f(x) = \sin\frac{\pi}{x}$ is not defined at $x = 0$.

Example 3

Find $\lim_{x \to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

Example 3

Find $\lim_{x\to 0} \sin\frac{\pi}{x}$. The function $f(x) = \sin\frac{\pi}{x}$ is not defined at $x = 0$.

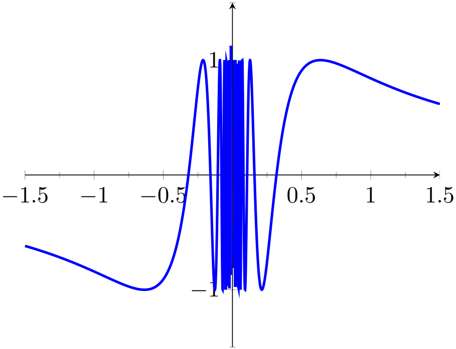
x	$\sin\frac{\pi}{x}$	x	$\sin\frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

We guess that $\lim_{x\to 0} \sin\frac{\pi}{x} = 0$.

Example 3

Find $\lim_{x \to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

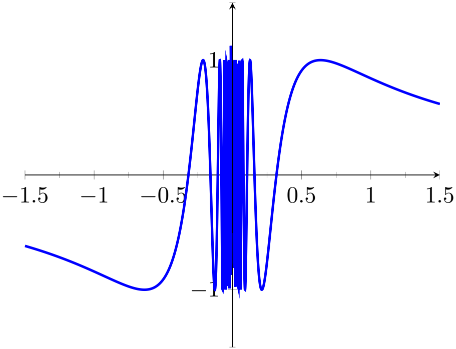


We guess that $\lim_{x \to 0} \sin \frac{\pi}{x} = 0$.

Example 3

Find $\lim_{x\to 0} \sin\frac{\pi}{x}$. The function $f(x) = \sin\frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin\frac{\pi}{x}$	x	$\sin\frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000

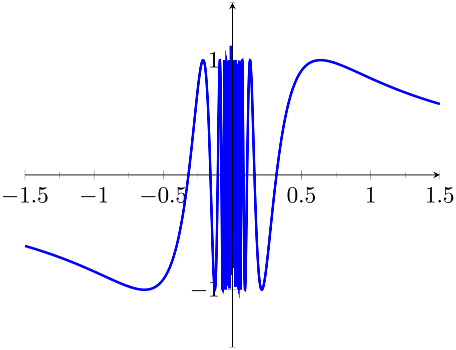


We guess that $\lim_{x\to 0} \sin\frac{\pi}{x} \neq 0$.

Example 3

Find $\lim_{x\to 0} \sin\frac{\pi}{x}$. The function $f(x) = \sin\frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin\frac{\pi}{x}$	x	$\sin\frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000



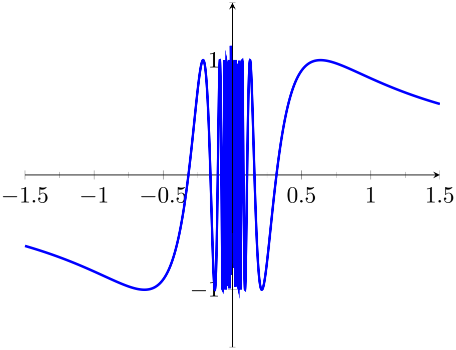
We guess that $\lim_{x\to 0} \sin\frac{\pi}{x} \neq 0$.

Reason: $\sin\frac{\pi}{0.01} = \sin(100\pi) = 0$, since $\sin\pi n = 0$.

Example 3

Find $\lim_{x\to 0} \sin\frac{\pi}{x}$. The function $f(x) = \sin\frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin\frac{\pi}{x}$	x	$\sin\frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000



We guess that $\lim_{x\to 0} \sin\frac{\pi}{x} \neq 0$.

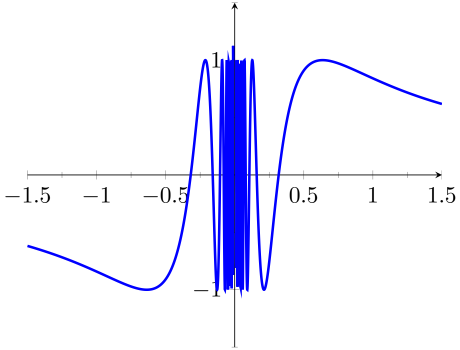
Reason: $\sin\frac{\pi}{0.01} = \sin(100\pi) = 0$, since $\sin\pi n = 0$.

However, $\sin(\pi n + \frac{\pi}{2}) = \pm 1$.

Example 3

Find $\lim\limits_{x\to 0} \sin\frac{\pi}{x}$. The function $f(x) = \sin\frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin\frac{\pi}{x}$	x	$\sin\frac{\pi}{x}$
-0.10000	0.000000	0.10000	0.000000
-0.01000	0.000000	0.01000	0.000000
-0.00100	0.000000	0.00100	0.000000
-0.00010	0.000000	0.00010	0.000000
-0.00001	0.000000	0.00001	0.000000



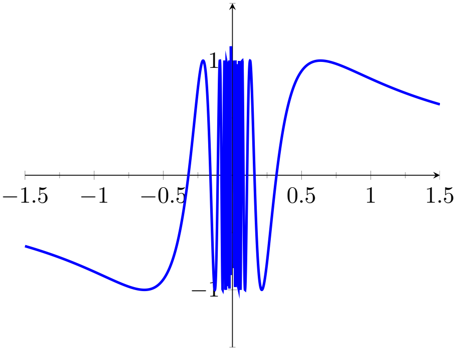
We guess that $\lim\limits_{x\to 0} \sin\frac{\pi}{x} \neq 0$.

Reason: $\sin\frac{\pi}{0.01} = \sin(100\pi) = 0$, since $\sin\pi n = 0$.
However, $\sin(\pi n + \frac{\pi}{2}) = \pm 1$. Solving $\pi n + \frac{\pi}{2} = \frac{\pi}{x}$ for x with $n = 100$ gives $x = 0.00995$ and $\sin\frac{\pi}{0.00995} = 0.997$

Example 3

Find $\lim_{x \to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at $x = 0$.

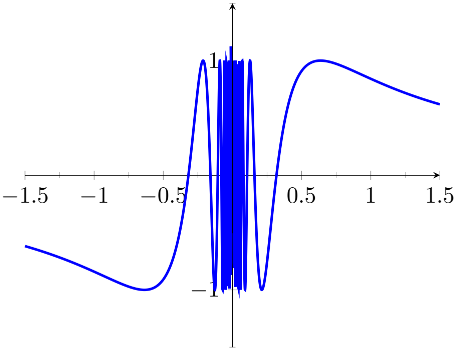
x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.49990	-0.0012569	0.50000	0.0000000
-0.02090	0.4626743	0.02100	-0.9308737
-0.00985	0.9974262	0.00995	0.9999688
-0.00063	0.8119380	0.00073	-0.4171936
-0.00011	-0.2817326	0.00011	0.2817326



Example 3

Find $\lim_{x \to 0} \sin \frac{\pi}{x}$. The function $f(x) = \sin \frac{\pi}{x}$ is not defined at $x = 0$.

x	$\sin \frac{\pi}{x}$	x	$\sin \frac{\pi}{x}$
-0.49990	-0.0012569	0.50000	0.0000000
-0.02090	0.4626743	0.02100	-0.9308737
-0.00985	0.9974262	0.00995	0.9999688
-0.00063	0.8119380	0.00073	-0.4171936
-0.00011	-0.2817326	0.00011	0.2817326



Therefore, $\lim_{x \to 0} \sin \frac{\pi}{x}$ does not exist..

One-Sided Limits

One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)

One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)

One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)

One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)

We say that $\lim_{x\to a^-} f(x) = L$ (left-hand limit) if we can make the values of $f(x)$ as close to L as we like by taking x to be any number smaller than a and sufficiently close to a.

One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)

We say that $\lim_{x \to a^-} f(x) = L$ (left-hand limit) if we can make the values of $f(x)$ as close to L as we like by taking x to be any number smaller than a and sufficiently close to a.

Similarly, $\lim_{x \to a^+} f(x) = L$ (right-hand limit) if we can make the values of $f(x)$ as close to L as we like by taking x to be any number greater than a and sufficiently close to a.

One-Sided Limits

For a number a we write

- $x \to a^-$ if x approaches a from the left (through values less than a)
- $x \to a^+$ if x approaches a from the right (through values greater than a)

We say that $\lim_{x \to a^-} f(x) = L$ (left-hand limit) if we can make the values of $f(x)$ as close to L as we like by taking x to be any number smaller than a and sufficiently close to a.

Similarly, $\lim_{x \to a^+} f(x) = L$ (right-hand limit) if we can make the values of $f(x)$ as close to L as we like by taking x to be any number greater than a and sufficiently close to a.

$\lim_{x \to a} f(x) = L$ *if and only if* $\lim_{x \to a^-} f(x) = L$ *and* $\lim_{x \to a^+} f(x) = L$

Example 1

Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x)$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 2} f(x)$

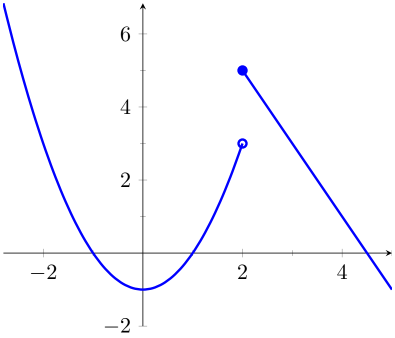
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x)$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 2} f(x)$



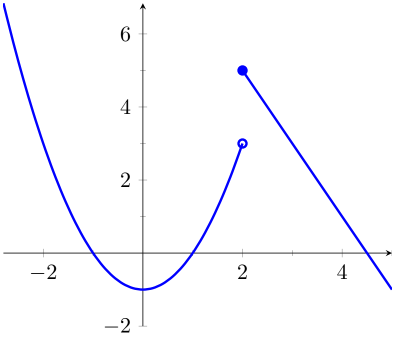
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x)$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 2} f(x)$



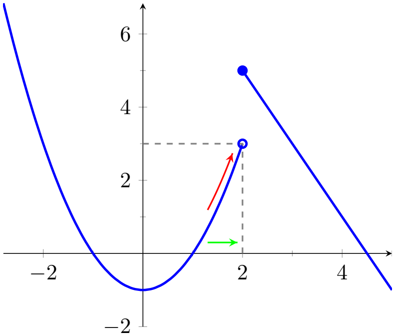
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x)$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 2} f(x)$



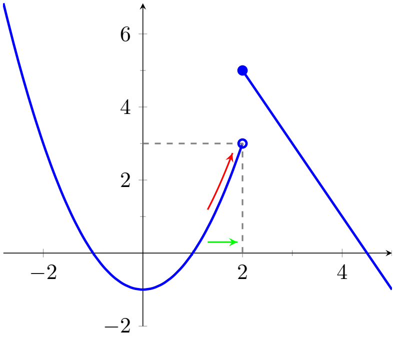
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x) = 3$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 2} f(x)$



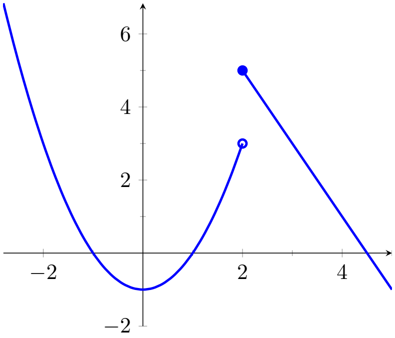
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x) = 3$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 2} f(x)$



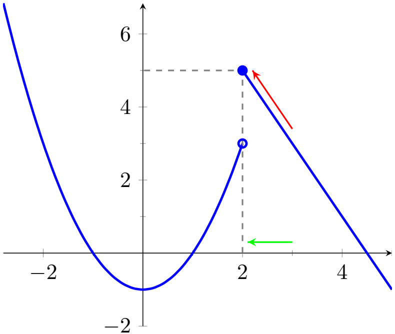
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x) = 3$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 2} f(x)$



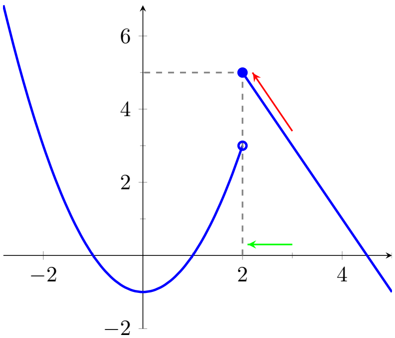
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x) = 3$
2. $\lim_{x \to 2^+} f(x) = 5$
3. $\lim_{x \to 2} f(x)$



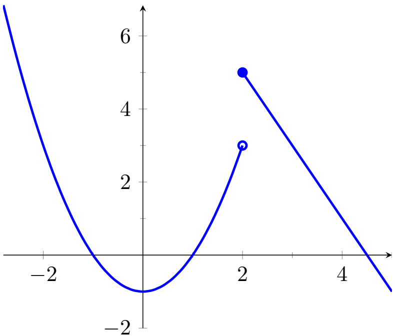
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x) = 3$
2. $\lim_{x \to 2^+} f(x) = 5$
3. $\lim_{x \to 2} f(x)$



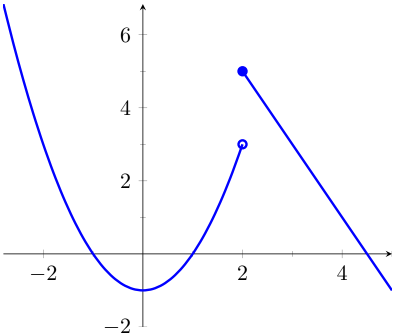
Example 1

For the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 9 - 2x & \text{if } x \geq 2 \end{cases}$$

state the value of each limit or show that it does not exist.

1. $\lim_{x \to 2^-} f(x) = 3$
2. $\lim_{x \to 2^+} f(x) = 5$
3. $\lim_{x \to 2} f(x)$ DNE, since $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$



Example 2

Investigate $\lim\limits_{x \to 0} \frac{|x|}{x}$.

Example 2

Investigate $\lim_{x \to 0} \frac{|x|}{x}$.

Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

Example 2

Investigate $\lim_{x \to 0} \frac{|x|}{x}$.

Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

When $x < 0$, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$.

Example 2

Investigate $\lim_{x \to 0} \frac{|x|}{x}$.

Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

When $x < 0$, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$.

When $x > 0$, we have $\frac{|x|}{x} = \frac{x}{x} = 1$.

Example 2

Investigate $\lim_{x \to 0} \frac{|x|}{x}$.

Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

When $x < 0$, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$.

When $x > 0$, we have $\frac{|x|}{x} = \frac{x}{x} = 1$.

Therefore, $\frac{|x|}{x} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$

Example 2

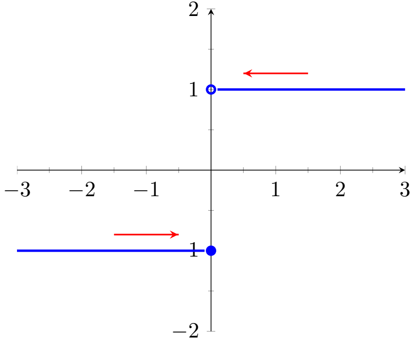
Investigate $\lim_{x\to 0} \frac{|x|}{x}$.

Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

When $x < 0$, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$.

When $x > 0$, we have $\frac{|x|}{x} = \frac{x}{x} = 1$.

Therefore, $\frac{|x|}{x} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$



Example 2

Investigate $\lim\limits_{x\to 0} \frac{|x|}{x}$.

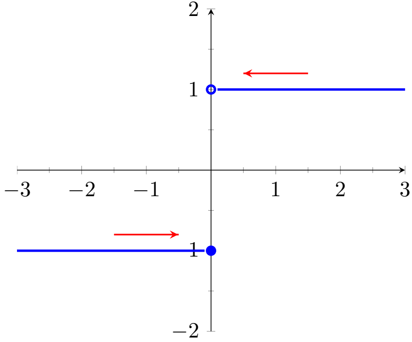
Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

When $x < 0$, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$.

When $x > 0$, we have $\frac{|x|}{x} = \frac{x}{x} = 1$.

Therefore, $\frac{|x|}{x} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$

$$\lim_{x\to 0^-} \frac{|x|}{x} = -1 \qquad \lim_{x\to 0^+} \frac{|x|}{x} = 1$$



Example 2

Investigate $\lim_{x\to 0} \frac{|x|}{x}$.

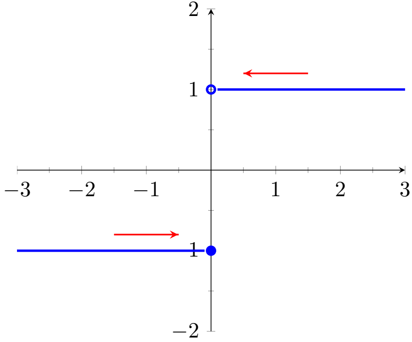
Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

When $x < 0$, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$.

When $x > 0$, we have $\frac{|x|}{x} = \frac{x}{x} = 1$.

Therefore, $\frac{|x|}{x} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$

$$\lim_{x\to 0^-} \frac{|x|}{x} = -1 \neq \lim_{x\to 0^+} \frac{|x|}{x} = 1$$



Example 2

Investigate $\lim_{x\to 0} \frac{|x|}{x}$.

Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

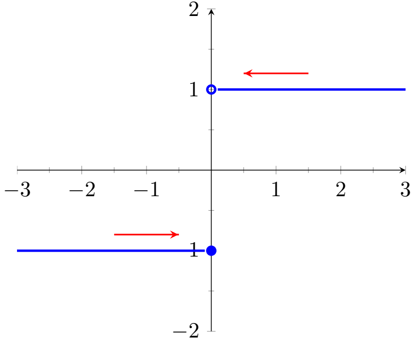
When $x < 0$, we have $\frac{|x|}{x} = \frac{-x}{x} = -1$.

When $x > 0$, we have $\frac{|x|}{x} = \frac{x}{x} = 1$.

Therefore, $\frac{|x|}{x} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$

$$\lim_{x\to 0^-} \frac{|x|}{x} = -1 \neq \lim_{x\to 0^+} \frac{|x|}{x} = 1$$

We conclude that $\lim_{x\to 0} \frac{|x|}{x}$ DNE.



Infinite Limits

Infinite Limits

$\lim_{x \to a} f(x) = \infty$ *means that the values of* $f(x)$ *can be made arbitrarily large by taking* x *to be any number sufficiently close (but not equal) to* a.

Infinite Limits

$\lim_{x \to a} f(x) = \infty$ *means that the values of* $f(x)$ *can be made arbitrarily large by taking* x *to be any number sufficiently close (but not equal) to* a.

For example,

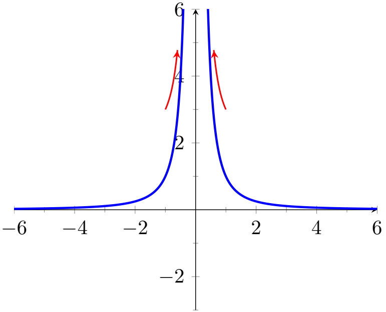
$$\lim_{x \to 0} \frac{1}{x^2}$$

Infinite Limits

$\lim_{x \to a} f(x) = \infty$ *means that the values of* $f(x)$ *can be made arbitrarily large by taking* x *to be any number sufficiently close (but not equal) to* a*.*

For example,

$$\lim_{x \to 0} \frac{1}{x^2}$$

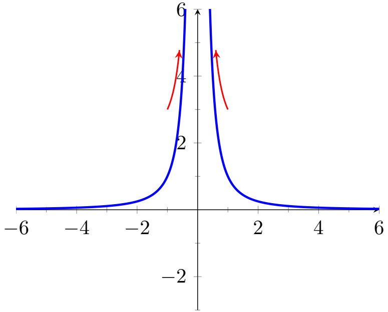


Infinite Limits

$\lim_{x \to a} f(x) = \infty$ *means that the values of* $f(x)$ *can be made arbitrarily large by taking* x *to be any number sufficiently close (but not equal) to* a.

For example,

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$



Infinite Limits

Infinite Limits

$\lim_{x \to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a.

Infinite Limits

$\lim_{x\to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a.

Example Find $\lim_{x\to 0^-} \frac{1}{x}$ and $\lim_{x\to 0^+} \frac{1}{x}$.

Infinite Limits

$\lim_{x \to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a.

Example Find $\lim_{x \to 0^-} \frac{1}{x}$ and $\lim_{x \to 0^+} \frac{1}{x}$.

If x is a small negative number, $\frac{1}{x}$ is a large negative number.

Infinite Limits

$\lim_{x\to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a*.*

Example Find $\lim_{x\to 0^-} \frac{1}{x}$ and $\lim_{x\to 0^+} \frac{1}{x}$.

If x is a small negative number, $\frac{1}{x}$ is a large negative number.

Therefore, $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.

Infinite Limits

$\lim_{x\to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a*.*

Example Find $\lim_{x\to 0^-} \frac{1}{x}$ and $\lim_{x\to 0^+} \frac{1}{x}$.

If x is a small negative number, $\frac{1}{x}$ is a large negative number.

Therefore, $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.

If x is a small positive number, $\frac{1}{x}$ is a large positive number.

Infinite Limits

$\lim_{x \to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a.

Example Find $\lim_{x \to 0^-} \frac{1}{x}$ and $\lim_{x \to 0^+} \frac{1}{x}$.

If x is a small negative number, $\frac{1}{x}$ is a large negative number.

Therefore, $\lim_{x \to 0^-} \frac{1}{x} = -\infty$.

If x is a small positive number, $\frac{1}{x}$ is a large positive number.

Therefore, $\lim_{x \to 0^+} \frac{1}{x} = \infty$.

Infinite Limits

$\lim_{x \to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a.

Example Find $\lim_{x \to 0^-} \frac{1}{x}$ and $\lim_{x \to 0^+} \frac{1}{x}$.

If x is a small negative number, $\frac{1}{x}$ is a large negative number.

Therefore, $\lim_{x \to 0^-} \frac{1}{x} = -\infty$.

If x is a small positive number, $\frac{1}{x}$ is a large positive number.

Therefore, $\lim_{x \to 0^+} \frac{1}{x} = \infty$.

Infinite Limits

$\lim_{x\to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a.

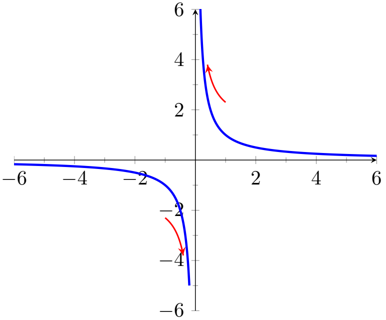
Example Find $\lim_{x\to 0^-} \frac{1}{x}$ and $\lim_{x\to 0^+} \frac{1}{x}$.

If x is a small negative number, $\frac{1}{x}$ is a large negative number.

Therefore, $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.

If x is a small positive number, $\frac{1}{x}$ is a large positive number.

Therefore, $\lim_{x\to 0^+} \frac{1}{x} = \infty$.



Infinite Limits

$\lim_{x\to a} f(x) = -\infty$ *means that the values of* $f(x)$ *can be made arbitrarily large negative by taking* x *to be any number sufficiently close (but not equal) to* a.

Example Find $\lim_{x\to 0^-} \frac{1}{x}$ and $\lim_{x\to 0^+} \frac{1}{x}$.

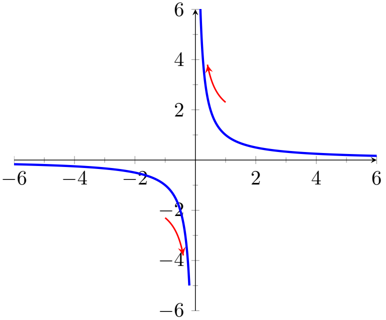
If x is a small negative number, $\frac{1}{x}$ is a large negative number.

Therefore, $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.

If x is a small positive number, $\frac{1}{x}$ is a large positive number.

Therefore, $\lim_{x\to 0^+} \frac{1}{x} = \infty$.

However, $\lim_{x\to 0} \frac{1}{x}$ DNE



THE END