Summation Notation

Tamara Kucherenko

Summation Notation

A standard way of writing sums in compact form uses the Greek letter Σ (sigma).

Definition

If $a_m, a_{m+1}, ..., a_n$ *are real numbers and* m *and* n *are integers such that* $m \leq n$, *then*

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + ... + a_n$$

The letter Σ stands for the sum and the notation $\sum_{i=m}^{n}$ tells us to start the summation at $i = m$ and end it at $i = n$.

The letter i is called the index of summation. Any other letter can be used as the index of the summation.

a_i is an expression depending on i which is called the general term of summation.

Examples

1. $\sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5 = 15$
2. $\sum_{j=4}^{6} j^2 = 4^2 + 5^2 + 6^2 = 77$
3. $\sum_{k=2}^{8} \frac{1}{k} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = \frac{1443}{840}$
4. $\sum_{i=1}^{7} 2 = 2 + 2 + 2 + 2 + 2 + 2 + 2 = 14$
5. Write in Σ-notation $\sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} = \sum_{i=3}^{9} \sqrt{i} = \sum_{j=1}^{7} \sqrt{j+2}$

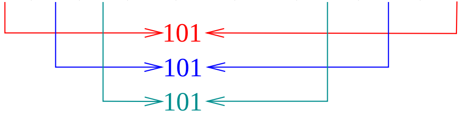
Examples

6. $$\sum_{i=1}^{n} 1 = \underbrace{1 + 1 + 1 + \ldots + 1}_{n \text{ times}} = n$$

7. $$\sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n$$

Gauss summation story:

$$1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100 = 101 \cdot 50 = 5050$$

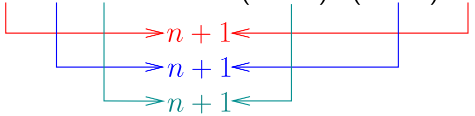


Examples

6. $\sum_{i=1}^{n} 1 = \underbrace{1+1+1+\ldots+1}_{n \text{ times}} = n$

7. $\sum_{i=1}^{n} i = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$

$$1+2+3+4+\ldots+(n-2)+(n-1)+n = (n+1)\cdot\frac{n}{2}$$



Summation Rules

Summation Formulas

1. $\sum_{i=1}^{n} 1 = n$ 2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ 3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ 4. $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

Summation Rules

1. $\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i$ (*c is a constant, does not depend on* i).
2. $\sum_{i=m}^{n} (a_i \pm b_i) = \sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i$

Evaluating Summations

Summation Formulas

1. $\sum_{i=1}^{n} 1 = n$
2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

Find the value of the sum

1. $\sum_{i=1}^{n}(7-5i) = \sum_{i=1}^{n} 7 - \sum_{i=1}^{n} 5i = 7\sum_{i=1}^{n} 1 - 5\sum_{i=1}^{n} i = 7n - 5\frac{n(n+1)}{2} = \frac{9n-5n^2}{2}$
2. $\sum_{i=1}^{n}(3+i)^2 = \sum_{i=1}^{n}(9+6i+i^2) = 9\sum_{i=1}^{n} 1+6\sum_{i=1}^{n} i+\sum_{i=1}^{n} i^2 = 9n+6\frac{n(n+1)}{2}+\frac{n(n+1)(2n+1)}{6}$
3. $\sum_{i=1}^{n}(i^3-i) = \sum_{i=1}^{n} i^3 - \sum_{i=1}^{n} i = \left[\frac{n(n+1)}{2}\right]^2 - \frac{n(n+1)}{2}$

THE END